1. Give an example of 3 events A, B, C which are pairwise independent but not independent. Hint: find an example where whether C occurs is completely determined if we know whether A occurred and whether B occurred, but completely undetermined if we know only one of these things.

Sol)

Let's consider a scenario with three events A, B, and C:

A: It is raining.

B: There are clouds in the sky.

C: People are carrying umbrellas.

We assume that A and B are independent, meaning that the occurrence of rain (A) does not affect the presence of clouds (B). Similarly, B and C are independent, meaning that the presence of clouds (B) does not affect whether people carry umbrellas (C).

However, if we know whether A and B occurred, we can determine whether C occurred. If it is raining (A) and there are clouds (B), it is highly likely that people will carry umbrellas (C). However, if it is not raining (A') and there are no clouds (B'), it is highly unlikely that people will carry umbrellas (C'). In this case, the occurrence of C depends on the joint occurrence of A and B.

Therefore, A, B, and C are pairwise independent but not independent.

1. A bag contains one marble which is either green or blue, with equal probabilities. A green marble is put in the bag (so there are 2 marbles now), and then a random marble is taken out. The marble taken out is green. What is the probability that the remaining marble is also green?

Sol)

Let's calculate the probability that the remaining marble is green given that the marble taken out is green:

P(Green marble taken out) = P(Green marble in the bag) \* P(Green marble taken out | Green marble in the bag)

= (1/2) \* 1

= 1/2

Now, we need to find the probability that the remaining marble is also green:

P(Remaining marble is green) = P(Green marble in the bag and Green marble taken out) / P(Green marble taken out)

= (1/2) / (1/2)

= 1

Therefore, the probability that the remaining marble is green is 1 (or 100%).